Effective range parameters for collisions between light ions in a microscopic model

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1. Introduction

- At very low energies:
  The phase shifts can be described with two or tree parameters: the scattering length, the effective range and the shape parameter.

- Direct calculation of the Effective Range parameters without any extrapolations

- Microscopic description

- Applications for different collisions

2. Microscopic Wave Functions and MRM Theory

The microscopic Hamiltonian:

$$H = \sum_{i=1}^{A} T_i + \sum_{i>j=1}^{A} V_{ij}$$

where

T_i: kinetic energy of nucleon i

V_{ij}: interaction between nucleons i and j

In the R-matrix method: two regions

In the internal region ($r \leq a$):
microscopic wave function decomposed in N basis functions:

$$\Psi_{int}^{lm} = \sum_{n=1}^{N} f_n^{Jl} \Phi_n^{lm}$$

where f_n^{lm}: generator functions
basis functions:

$$\Phi_n^{lm} = \mathcal{A}\phi_1\phi_2\Gamma_{ln}(r)Y_l^m(\Omega_r) \qquad (1)$$

where $\mathcal{A}$: antisymmetrizor operator
$\mathbf{r} = (r, \Omega)$: relative coordinate
Γ_{ln}: relative function
ϕ_1 and ϕ_2: internal wave functions

In the external region ($r \geq a$):
the antisymmetrisation is neglected

$$\Psi_{ext}^{lm} = \phi_1\phi_2[\mathcal{F}_l(E,r) + D_l(E)\mathcal{G}_l(E,r)]$$

$$\Psi_{ext}^{lm} = \phi_1 \phi_2 [\mathcal{F}_l(E,r) + D_l(E)\mathcal{G}_l(E,r)]$$

NEUTRAL CASE

$$D_l(E) = k^{-2l-1} \tan \delta_l(E)$$

$$\mathcal{F}_l(E,r) = k^{-l} r j_l(kr)$$

$$\mathcal{G}_l(E,r) = k^{l+1} r n_l(kr)$$

CHARGED CASE

$$D_l(E) = \frac{2}{\pi} \exp(2\pi\eta) \tan \delta_l(E)$$

$$\mathcal{F}_l(E,r) = k^{-1/2} \exp(\pi\eta) F_l(kr)$$

$$\mathcal{G}_l(E,r) = \frac{\pi}{2} k^{-1/2} \exp(-\pi\eta) G_l(kr)$$

η: Sommerfeld parameter

F_l and G_l: regular and irregular Coulomb functions

δ_l: phase shift

Continuity of the wave function:

$$D_l(E) = -\frac{\mathcal{F}_l(E,a) - aR_l(E)\partial\mathcal{F}_l(E,a)/\partial a}{\mathcal{G}_l(E,a) - aR_l(E)\partial\mathcal{G}_l(E,a)/\partial a}$$

R-matrix:

$$R_l(E) = \frac{\hbar^2 a}{2\mu} \sum_{n,n'=1}^{N} \Gamma_{ln}(a) \Big[(C^l - EN^l)^{-1} \Big]_{n',n} \Gamma_{l'n'}(a)$$

where:

$$\Big[C^l - EN^l \Big]_{n',n} = \langle \Phi_{n'}^{lm} | H + \mathcal{L} - E | \Phi_n^{lm} \rangle_{int}$$

Bloch operator:

$$\mathcal{L} = \frac{\hbar^2}{2\mu a} \delta(r-a) \left(\frac{\partial}{\partial r} \right) r$$

3. Effective Range Parameters

(NEUTRAL CASE)

$$\frac{1}{D_l(E)} = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 - P_l r_l^3 k^4 + O(k^6)$$

a_l: scattering length

$$a_l = -D_l(0)$$

r_l: effective range

$$r_l = -\frac{\hbar^2}{\mu a_l^2} D_l'(0)$$

P_l: shape parameter

$$P_l = \frac{a_l}{4 r_l} + \frac{1}{8 r_l} \left(\frac{\hbar^2}{\mu a_l r_l} \right)^2 D_l''(0)$$

$$D_l(E) = D_l(0) + D_l'(0)E + 1/2 D_l''(0)E^2$$

$$D_l(0) = -\frac{\mathcal{F}_l^0(a) - aR_l^0 d\mathcal{F}_l^0(a)/da}{\mathcal{G}_l^0(a) - aR_l^0 d\mathcal{G}_l^0(a)/da}$$

$D_l'(0)$ and $D_l''(0)$ can easily be calculated

$D_l'(0)$ and $D_l''(0)$ depend on $R_l(0)$, $R_l'(0)$ and $R_l^{''}(0)$

$$R_l(E) = R_l(0) + R_l'(0)E + 1/2 R_l''(0)E^2$$

$$\mathcal{F}_l^0(r) = \frac{r^{l+1}}{(2l+1)!!} \qquad \mathcal{F'}_l^0(r) = \left(-\frac{\mu}{\hbar^2}\right)\frac{r^{l+3}}{(2l+3)!!}$$

$$\mathcal{G}_l^0(r) = (2l-1)!!\, r^{-l} \qquad \mathcal{G'}_l^0(r) = \left(\frac{\mu}{\hbar^2}\right)(2l-3)!!\, r^{-l+2}$$

4. Applications

Comparison:

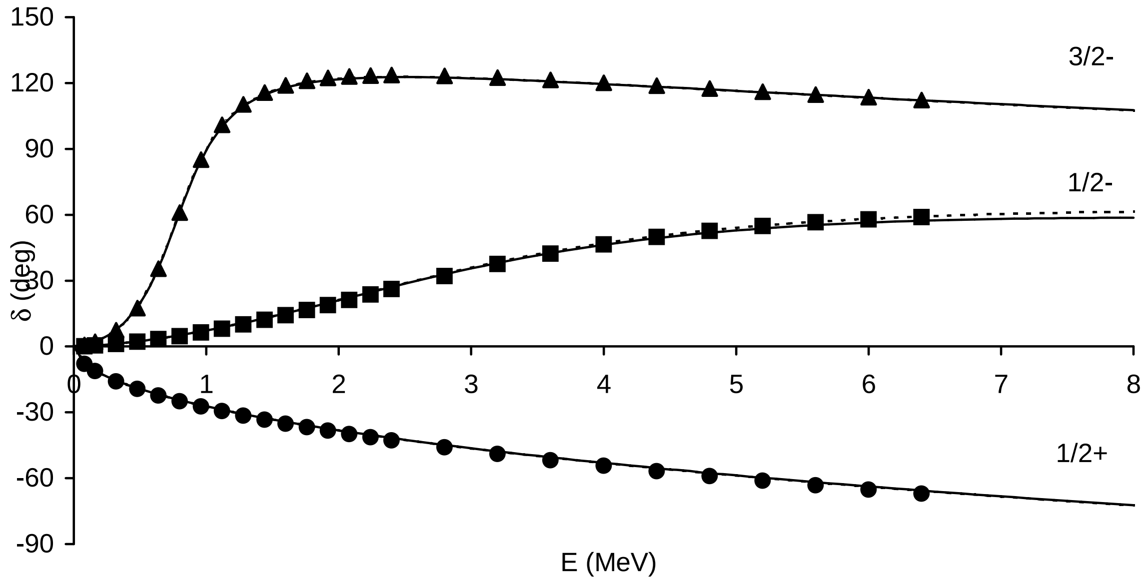
- experimental phase shift $\delta_l^{exp}(E)$
- microscopic phase shift $\delta_l(E)$
- phase shift $\delta_l^{ER}(E)$

$$\delta_l^{ER}(E) = \mathrm{atan}\,\left(k^{2l+1} D_l(E)\right)$$

in the neutral case

$$\frac{1}{D_l(E)} = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 - P_l r_l^3 k^4 + O(k^6)$$

$\alpha + n$ phase shifts



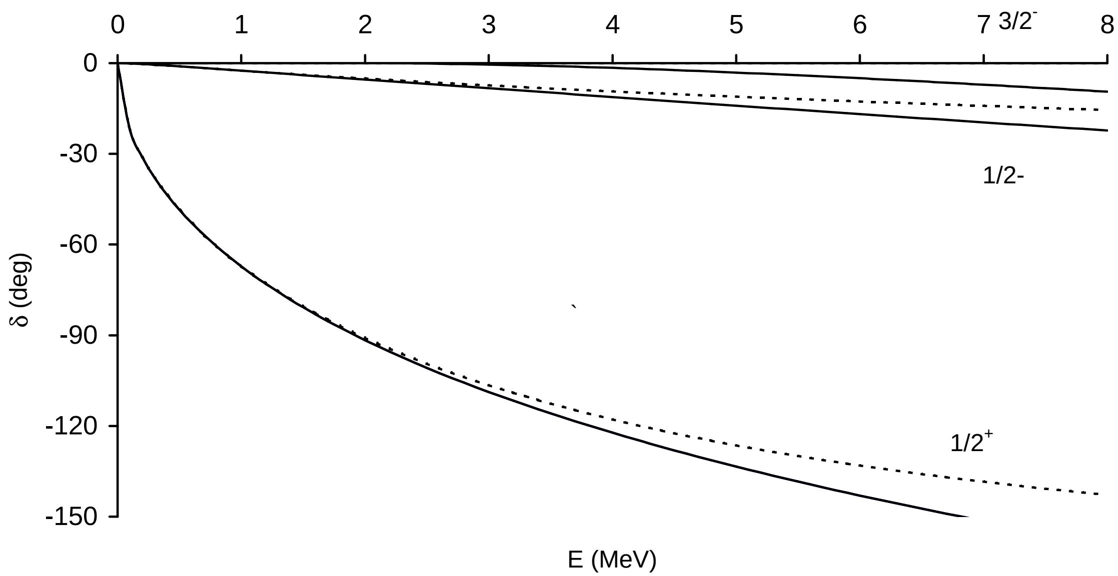
Minnesota: $u = 0.960$, $S_0 = 37.60$

Full curve: Microscopic calculation $\delta_l(E)$

Dashed curve: Effective range expansion $\delta_l^{ER}(E)$

Circles, triangles and squares: Experimental values $\delta_l^{exp}(E)$

^{16}O+n phase shifts



Minnesota: $u = 0.924$, $S_0 = 34.40$
Full curve: Microscopic calculation $\delta_l(E)$
Dashed curve: Effective range expansion $\delta_l^{ER}(E)$

Neutral Case

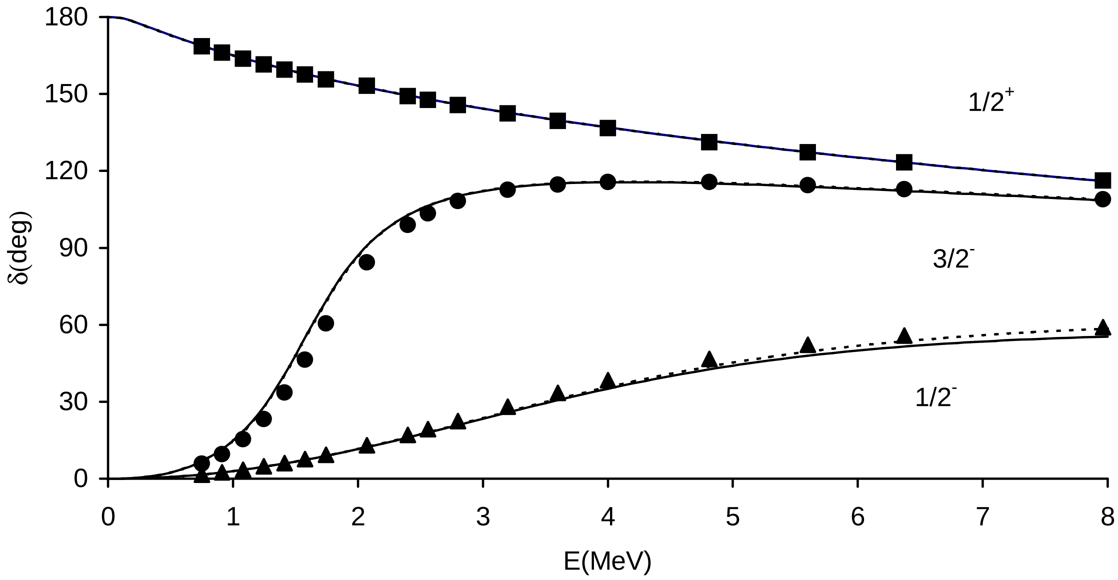
Effective range coefficients for neutron reactions

collision	J^π	a_l(fm^{2l+1})	r_l (fm^{-2l+1})
α + n	$1/2^+$	2.45	1.43
	$1/2^-$	-15.18	-0.42
	$3/2^-$	-65.43	-0.84
^{16}O + n	$1/2^+$	5.76	3.45
	$1/2^-$	7.14	-3.74
	$3/2^-$	0.46	-304

$\alpha + n$: a_0^{exp}=2.456 $\pm$ 0.002 fm

^{16}O + n: a_0^{exp}=5.464 $\pm$ 0.005 fm

α+p phase shifts



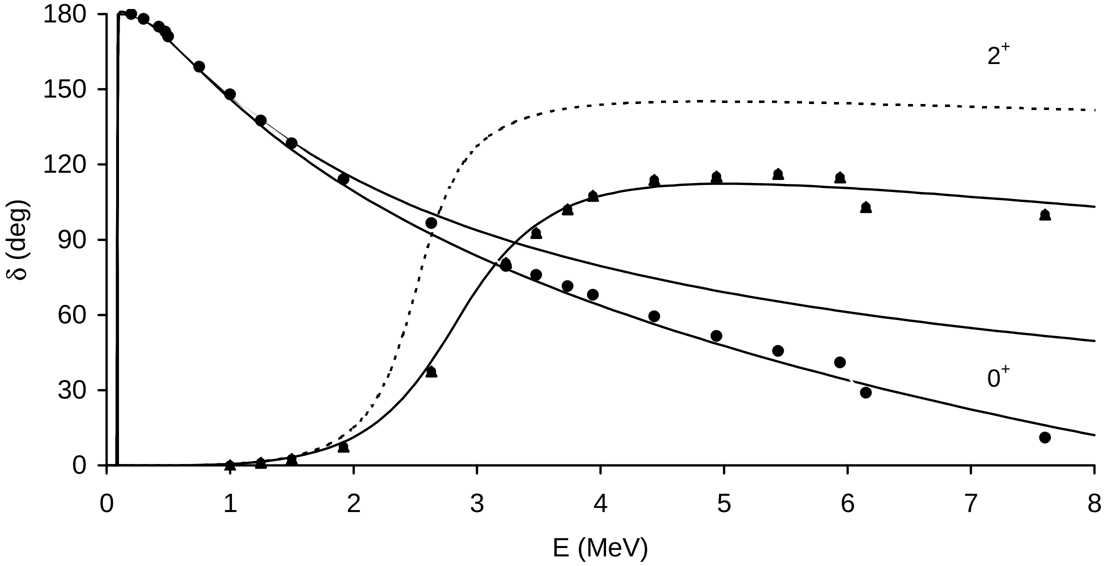
Minnesota: $u = 0.960$, $S_0 = 37.00$

Full curve: Microscopic calculation $\delta_l(E)$

Dashed curve: Effective range expansion $\delta_l^{ER}(E)$

Circles, triangles and squares: Experimental values $\delta_l^{exp}(E)$

$\alpha + \alpha$ phase shifts



Minnesota: $u = 0.9474$

Full curve: Microscopic calculation $\delta_l(E)$

Dashed curve: Effective range expansion $\delta_l^{ER}(E)$

Circles, triangles and squares: Experimental values $\delta_l^{exp}(E)$

Charged case

Effective range coefficients

collision	J^π	$a_l(\text{fm}^{2l+1})$	r_l (fm^{-2l+1})
$\alpha + \alpha$	0^+	-2388.35	1.11
	2^+	-18.99	0.07
$^{16}\text{O} + \text{p}$	$1/2^+$	4676.72	1.18
$\alpha + \text{p}$	$1/2^+$	4.87	1.26
	$1/2^-$	-16.48	-0.04
	$3/2^-$	-47.31	-0.37

5. CONCLUSIONS

- Microscopic calculation
- Evaluation of the effective range parameters for different partial wave
- Results without any extrapolation
- Good agreement with experiment